Statistical measure of complexity in compact stars with global charge neutrality

The Structure and Signals of Neutron Stars, Firenze, 2014

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Outline

Context

- This presentation is a sequence of the work started by de Avellar and Horvath.
- We now extend the technique in order to cover a bigger range of EoSs.
- Objective: to address the composition of neutron stars: hierarchy of EoSs through information theory techniques.

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Entropy, complexity and disequilibrium in compact stars

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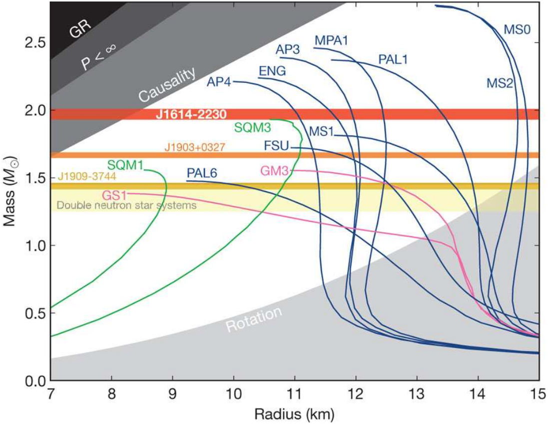
ABSTRACT

We used the statistical measurements of information entropy, disequilibrium and complexity to infer a hierarchy of equations of state for two types of compact stars from the broad class of neutron stars, namely, with hadronic composition and with strange quark composition. Our results show that, since order costs energy, Nature would favor the exotic strange stars even though the question of how to form the strange stars cannot be answered within this approach.

Mass radius diagram reflects the composition



How much information does the different EoSs possess? Which one, then, would be more likely to be realized in Nature?

Key concepts

- Complexity: A measure of how far the object is from an ideal system, in a sense that, in physics, we always begin with ideal systems as the simplest systems possible.
- Information: what we can get from observing the occurrence of an event (how surprising, or unexpected or what else). Defined in terms of the probability of an event to occur.
- Disequilibrium: The information alone is not enough to define complexity. We define then the *disequilibrium* as the distance to the equiprobability.

Key concepts

Complexity

Encodes order and disorder (or the self-organization of a system). It quantifies the locus of the studied system between two ideal systems, extremes in all aspects and opposites as well:

- Perfect crystal: zero complexity by definition; strict symmetry rules $\Rightarrow$ probability density centered around the prevailing state of perfect symmetry $\Rightarrow$ minimal information. Completely ordered.
- Ideal gas: zero complexity by definition; accessible states are equiprobable $\Rightarrow$ maximal information. Totally disordered.

Key concepts

Information

- In mathematical terms defined by Shannon, information is given by

$$I(p) = -log_b(p) \tag{1}$$

  for some probability p and basis b (that gives the unit).
- As a example, $b = 2$ give us *bits*, $b = e$ give us *nats*.
- With regard to physical systems, information can be obtained by the probability distribution of the allowed states in the system.
- Applying the theory, flipping a fair coin once give you $-log_2(1/2) = 1$ bit of information.

The continuous case - expressions

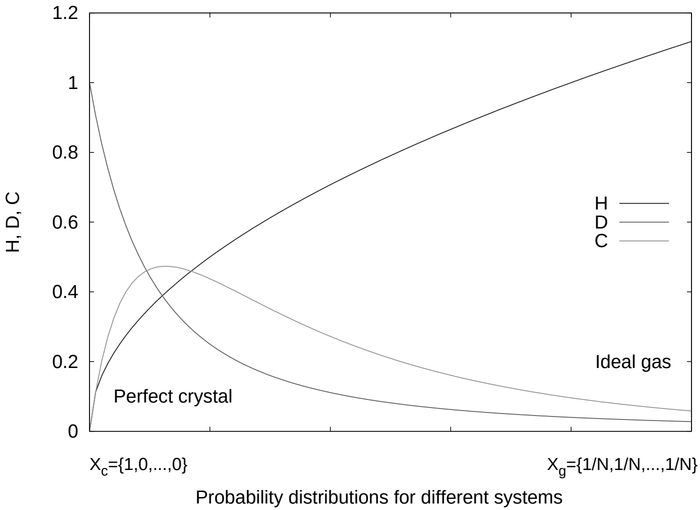
In the continuous case with large number of states N we get:

$$H = -\int p(x) log_b[p(x)] dx \tag{2}$$

$$D = \int p^2(x) dx \tag{3}$$

$$C \equiv e^H \times D. \tag{4}$$

Putting it all together - the intuition plot



Applying the theory to Compact Stars

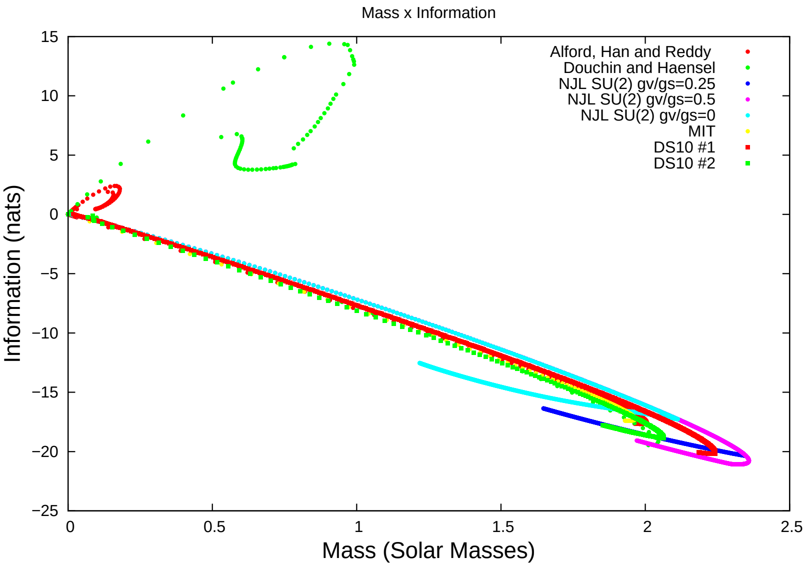
In the present work, we study the information content of the following equations of state:

- Douchin and Haensel (Astron. Astrophys. 380, 151)
- Alford, Han and Reddy (arXiv:1111.3937)
- DS10
- MIT bag model
- NJL SU(2)

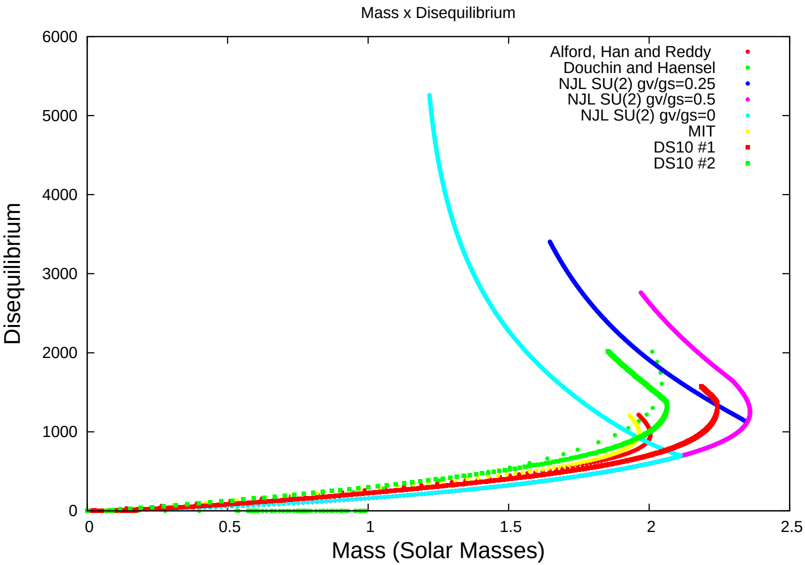
How does the composition affect the measures of these quantities? But first, what should we adopt as $p(x)$?

Applying the theory to Compact Stars

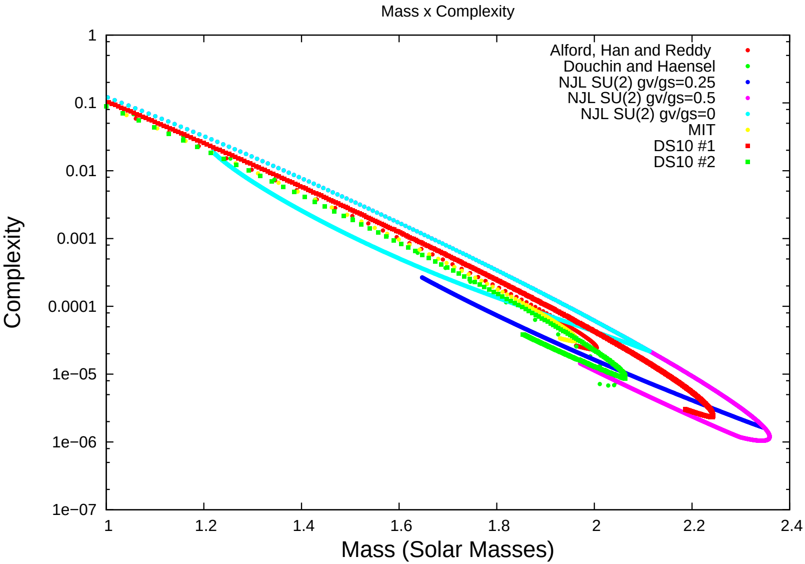
Following the works of Sañudo and Pacheco *(Phys. Lett. A 373 (2009) 807)* and Chatzisavvas et al *(Phys. Lett. A 373, (2009) 3901)*, we use the density profile of the star, after integrating the TOV equations.


Mass x Information
Alford, Han and Reddy
Douchin and Haensel
NJL SU(2) gv/gs=0.25
NJL SU(2) gv/gs=0.5
NJL SU(2) gv/gs=0
MIT
DS10 #1
DS10 #2
Information (nats)
Mass (Solar Masses)
15
10
5
0
−5
−10
−15
−20
−25
0
0.5
1
1.5
2
2.5

Mass vs Disequilibrium



Mass vs Complexity



Summary

Summary of the results:

- Hadronic stars (Douchin and Haensel): low complexity, ordered systems, tend to the side of the perfect crystal;
- Quark stars: low complexity, less ordered systems, more distant from perfect crystal;
- If order costs energy, then nature should favour exotic strange quark stars.

Next steps

Some perspectives regarding the development of these concepts:

- Recently, the local charge neutrality has been discussed, and some studies, like the one from Rotondo et al (arXiv:1308.3519) claims the impossibility of such configuration. We are going to apply information theory in order to infer which configuration is more favorable in Nature, but our preliminary studies show that local charge neutrality is preferred over global(arXiv:1308.3519).
- Improve the very concept of $p(x)$ used here in order to make it compatible to the analogous in information theory;
- Link to the thermodynamics and the gravitational collapse.

The End

Thanks!